ACT-02-10, MIFP-10-07

Generalizing Minimal Supergravity

Tianjun Li[1,2] and Dimitri V. Nanopoulos[1,3,4]

[1] *George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA*

[2] *Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China*

[3] *Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA*

[4] *Academy of Athens, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece*

Abstract

In Grand Unified Theories (GUTs), the Standard Model (SM) gauge couplings need not be unified at the GUT scale due to the high-dimensional operators. Considering gravity mediated supersymmetry breaking, we study for the first time the generic gauge coupling relations at the GUT scale, and the general gaugino mass relations which are valid from the GUT scale to the electroweak scale at one loop. We define the index k for these relations, which can be calculated in GUTs and can be determined at the Large Hadron Collider and the future International Linear Collider. Thus, we give a concrete definition of the GUT scale in these theories, and suggest a new way to test general GUTs at future experiments. We also discuss five special scenarios with interesting possibilities. With our generic formulae, we present all the GUT-scale gauge coupling relations and all the gaugino mass relations in the $SU(5)$ and $SO(10)$ models, and calculate the corresponding indices k. Especially, the index k is $5/3$ in the traditional $SU(5)$ and $SO(10)$ models that have been studied extensively so far. Furthermore, we discuss the field theory realization of the $U(1)$ flux effects on the SM gauge kinetic functions in F-theory GUTs, and calculate their indices k as well.

PACS numbers: 11.10.Kk, 11.25.Mj, 11.25.-w, 12.60.Jv

I. INTRODUCTION

Supersymmetry provides a natural solution to the gauge hierarchy problem in the Standard Model (SM). In supersymmetric SMs with R parity under which the SM particles are even while their supersymmetric partners are odd, the gauge couplings for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gauge symmetries are unified at about 2×10^{16} GeV [1], the lightest supersymmetric particle (LSP) like the neutralino can be the cold dark matter candidate [2, 3], and the electroweak precision constraints can be evaded, etc. In particular, gauge coupling unification [1] strongly suggests Grand Unified Theories (GUTs), which explain the quantum numbers of the SM fermions and charge quantization. Thus, the great challenge is how to test the supersymmetric GUTs at the Large Hadron Collider (LHC), the future International Linear Collider (ILC), and other experiments.

In the supersymmetric SMs, supersymmetry is broken in the hidden sector, and then its breaking effects are mediated to the SM observable sector. However, the relations between the supersymmetric particle (sparticle) spectra and the fundamental theories can be very complicated and model dependent. Interestingly, comparing to the supersymmetry breaking soft masses for squarks and sleptons, the gaugino masses have the simplest form and appear to be the least model dependent. With gravity mediated supersymmetry breaking in GUTs, we have a universal gaugino mass $M_{1/2}$ at the GUT scale, which is called the minimal supergravity (mSUGRA) scenario [4]. Thus, we have the gauge coupling relation and the gaugino mass relation at the GUT scale $M_{\rm GUT}$:

$$\frac{1}{\alpha_3} ~=~ \frac{1}{\alpha_2} ~=~ \frac{1}{\alpha_1} ~, \tag{1}$$

$$\frac{M_3}{\alpha_3} ~=~ \frac{M_2}{\alpha_2} ~=~ \frac{M_1}{\alpha_1} ~, \tag{2}$$

where α_3, α_2, and $\alpha_1 \equiv 5\alpha_Y/3$ are gauge couplings respectively for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge symmetries, and M_3, M_2, and M_1 are the masses for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauginos, respectively. Interestingly, $1/\alpha_i$ and M_i/α_i satisfy the same equation $x_3 = x_2 = x_1$ at the GUT scale, which will be proved as a general result. Because M_i/α_i are constant under one-loop renormalization group equation (RGE) running, we obtain that the above gaugino mass relation in Eq. (2) is valid from the GUT scale to the electroweak scale at one loop. Note that the two-loop RGE running effects on gaugino masses are very small, thus, we can test this gaugino mass relation at the LHC and ILC where the gaugino masses can be measured [5, 6]. However, the SM gauge couplings in GUTs need not be unified at the GUT scale after the GUT gauge symmetry breaking since the high-dimensional operators will contribute to the different gauge kinetic terms for the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge symmetries [7–11]. Furthermore, we will have non-universal gaugino masses at the GUT scale as well [10–18]. In particular, in the GUTs with large number of fields, the renormalization effects significantly decrease the scale at which quantum gravity becomes strong, so, these high-dimensional operators are indeed important and need to be considered seriously [19]. Therefore, the key question is whether we still have the gaugino mass relations that can be tested at the LHC and ILC. It is amusing to notice that the first systematic studies for $SU(5)$ models in the framework of $N = 1$ supergravity for non-universal gauge couplings and gaugino masses at the GUT scale were done twenty-five years ago [10].

On the other hand, in F-theory model building [20–31], the GUT gauge fields are on the observable seven-branes which wrap a del Pezzo n surface dP_n for the extra four space dimensions. The SM fermions and Higgs fields are on the complex codimension-one curves (two-dimensional real subspaces) in dP_n, and the SM fermion Yukawa couplings arise from the intersections of the SM fermion and Higgs curves. A brand new feature is that the $SU(5)$ gauge symmetry can be broken down to the SM gauge symmetry by turning on $U(1)_Y$ flux [22, 23, 29], and the $SO(10)$ gauge symmetry can be broken down to the $SU(5) \times U(1)_X$ and $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetries by turning on the $U(1)_X$ and $U(1)_{B-L}$ fluxes, respectively [22, 23, 25, 26, 28, 29]. It has been shown that the gauge kinetic functions receive the corrections from $U(1)$ fluxes [24, 27–29]. Thus, whether we can test F-theory GUT at the LHC and ILC is another interesting question [31].

In this paper, we consider the generalization of the mSUGRA (GmSUGRA). In GUTs with gravity mediated supersymmetry breaking, we study for the first time the generic gauge coupling relations at the GUT scale, and the general gaugino mass relations which are valid from the GUT scale to the electroweak scale at one loop. Interestingly, the gauge coupling relations and the gaugino mass relations at the GUT scale are given by the same equation. In other words, $1/\alpha_i$ and M_i/α_i satisfy the same equation at the GUT scale respectively for the gauge coupling relations and the gaugino mass relations. Thus, we define the index k for these relations, which can be calculated in GUTs and can be determined at the LHC and ILC. Therefore, we present a concrete definition of the GUT scale in these theories, and suggest a new way to test general GUTs at the LHC and ILC. Also, we discuss five special scenarios with interesting possibilities. With our generic formulae, we present all the GUT-scale gauge coupling relations and all the gaugino mass relations in the $SU(5)$ and $SO(10)$ models, and calculate the corresponding indices. Especially, in the traditional $SU(5)$ and $SO(10)$ models that have been studied extensively thus far, the index k is 5/3, which was first pointed out for $SU(5)$ models in Ref. [10]. Moreover, we give the field theory realization of the $U(1)$ flux effects on the SM gauge kinetic functions in F-theory GUTs. We find that in the $SU(5)$ and $SO(10)$ models respectively with $U(1)_Y$ and $U(1)_{B-L}$ fluxes, the index k is 5/3, while in the $SO(10)$ models with $U(1)_X$ flux, the gauge coupling relation and the gaugino mass relation are the same as these in the mSUGRA. Furthermore, in four-dimensional GUTs, the GUT gauge symmetry breaking may also affect the supersymmetry breaking scalar masses, trilinear soft terms as well as the SM fermion Yukawa couplings, which will be studied elsewhere [32].

II. GAUGE COUPLING RELATIONS AND GAUGINO MASS RELATIONS

After the GUT gauge symmetry breaking, we can parametrize the gauge kinetic functions f_3, f_2 and f_1 respectively for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gauge symmetries at the GUT scale as follows

$$f_i \;=\; \sum_m a'_m \tau_m + \epsilon \left(\sum_n a_{in} S_n \right) \;, \tag{3}$$

where the first term is the original GUT gauge kinetic function, and the second term arise from the GUT gauge symmetry breaking. ϵ is a small paramter close to the ratio between the GUT Higgs vacuum expectation value (VEV) and the fundamental scale M_*. τ_m and S_n are the hidden sector fields whose F-terms may break supersymmetry. In particular, for $a_{1n} = a_{2n} = a_{3n}$, the gauge coupling relation at the GUT scale and the gaugino mass relation are the same as these in the mSUGRA.

Theorem. If there exist three real numbers b_i such that $\sum_{i=1}^3 b_i f_i = 0$, we have the following gauge coupling relation at the GUT scale

$$\frac{b_3}{\alpha_3} + \frac{b_2}{\alpha_2} + \frac{b_1}{\alpha_1} = 0 \;. \tag{4}$$

Using one-loop RGE running, we have the following gaugino mass relation which is renormalization scale invariant from the GUT scale to the electroweak scale at one loop

$$\frac{b_3 M_3}{\alpha_3} + \frac{b_2 M_2}{\alpha_2} + \frac{b_1 M_1}{\alpha_1} = 0 \ . \tag{5}$$

Proof. Because $f_i = 1/(4\pi\alpha_i)$, the gauge coupling relation in Eq. (4) at the GUT scale is obtained automatically.

From $\sum_{i=1}^{3} b_i f_i = 0$, we have

$$\sum_{i=1}^{3} b_i \ = \ 0 \ , \quad \sum_{i=1}^{3} b_i a_{in} \ = \ 0 \ . \tag{6}$$

Assuming that the F-terms of τ_m and S_n break supersymmetry, we obtain the ratios between the gaugino masses and gauge couplings

$$\frac{M_i}{\alpha_i} \ = \ 4\pi \left[\sum_m a'_m F^{\tau_m} + \epsilon \left(\sum_n a_{in} F^{S_n} \right) \right] \ . \tag{7}$$

Using Eq. (6), we obtain the gaugino mass relation given in Eq. (5) at the GUT scale. Because M_i/α_i are invariant under one-loop RGE running, we prove the theorem. The gaugino mass relation will have very small deviation due to the two-loop RGE running [31].

Interestingly, the GUT-scale gauge coupling relation in Eq. (4) and the gaugino mass relation in Eq. (5) give the same equation as follows

$$b_3 x_3 + b_2 x_2 + b_1 x_1 \ = \ 0 \ . \tag{8}$$

In other words, $1/\alpha_i$ and M_i/α_i at the GUT scale satisfy the same equation respectively for the gauge coupling relation and the gaugino mass relation. Thus, we can define the GUT scale in these theories:

Definition. The GUT scale is the scale at which $1/\alpha_i$ and M_i/α_i satisfy the same equation respectively for the gauge coupling relation and the gaugino mass relation.

For simplicity, we consider two supersymmetry breaking fields τ and S. The generic gauge kinetic function can be parametrized as follows

$$f_i \ = \ \tau + \epsilon a_i S \ . \tag{9}$$

If $a_1 = a_2 = a_3$, similar to the mSUGRA, we obtain the GUT-scale gauge coupling relation in Eq. (1) and the gaugino mass relation in Eq. (2).

If there exists at least one $a_i \neq a_j$ for $i \neq j$, we obtain the generic solution for b_i up to a scale factor

$$b_1 \;=\; a_2 - a_3 \;, \quad b_2 \;=\; a_3 - a_1 \;, \quad b_3 \;=\; a_1 - a_2 \;. \tag{10}$$

Using our theorem, we obtain the gauge coupling relation at the GUT scale

$$\frac{a_1 - a_2}{\alpha_3} + \frac{a_3 - a_1}{\alpha_2} + \frac{a_2 - a_3}{\alpha_1} = 0 \;. \tag{11}$$

In addition, we obtain the gaugino mass relation which is valid from the GUT scale to the electroweak scale under one-loop RGE running

$$\frac{(a_1 - a_2)M_3}{\alpha_3} + \frac{(a_3 - a_1)M_2}{\alpha_2} + \frac{(a_2 - a_3)M_1}{\alpha_1} = 0 \;. \tag{12}$$

Except the mSUGRA, we always have $a_1 \neq a_2$ or $a_1 \neq a_3$ in GmSUGRA in the following discussions. Thus, we can rewrite the GUT-scale gauge coupling relation and the gaugino mass relation as follows

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_3} \;=\; k\left(\frac{1}{\alpha_1} - \frac{1}{\alpha_3}\right) \;, \tag{13}$$

$$\frac{M_2}{\alpha_2} - \frac{M_3}{\alpha_3} \;=\; k\left(\frac{M_1}{\alpha_1} - \frac{M_3}{\alpha_3}\right) \;, \tag{14}$$

where k is the index of these relations, and is defined as follows

$$k \;\equiv\; \frac{a_2 - a_3}{a_1 - a_3} \;. \tag{15}$$

Because M_i/α_i are renormalization scale invariant under one-loop RGE running and can be calculated from the LHC and ILC experiments, k can be determined at the low energy as well. Therefore, we can test GUTs since its k can be calculated. Although k is not well defined in the mSUGRA, we symbolically define the index k for mSUGRA as $k = 0/0$. In other words, for $k = 0/0$, we have the gauge coupling relation at the GUT scale given by Eq. (1), and the gaugino mass relation given by Eq. (2). In addition, the concrete GUT scale can be redefined as follows:

Definition. The GUT scale is the scale at which the gauge coupling relation and the gaugino mass relation have the same index k.

Because the GUT gauge couplings should be positive and finite, we obtain that $\mathrm{Re}\tau > 0$. Let us consider five special cases in the following:

(1) Re$S \neq 0$, $F^\tau \neq 0$, $F^S = 0$.

In this case, the gauge coupling relation at the GUT scale is still given by Eq. (11) or Eq. (13). However, the gaugino mass relation is given by the mSUGRA gaugino mass relation in Eq. (2). This implies that even if we obtain the mSUGRA gaugino mass relation at the LHC and ILC, we may still have the non-unified SM gauge couplings at the GUT scale. Unfortunately, we can not calculate k in this case at the LHC and ILC.

(2) Re$S \neq 0$, $F^\tau = 0$, $F^S \neq 0$.

This case has been studied carefully in Refs. [12–18]. In this case, the gauge coupling relation at the GUT scale is still given by Eq. (11) or Eq. (13), and the gaugino mass relation is given by Eq. (12) or Eq. (14). In particular, for $a_i \neq 0$ we obtain the gaugino mass relation

$$\frac{M_3}{a_3\alpha_3} ~=~ \frac{M_2}{a_2\alpha_2} ~=~ \frac{M_1}{a_1\alpha_1} ~. \tag{16}$$

(3) Re$S = 0$, $F^\tau \neq 0$, $F^S \neq 0$.

In this case, the gauge coupling relation at the GUT scale is given by the mSUGRA gauge coupling relation in Eq. (1), while the gaugino mass relation is given by Eq. (12) or Eq. (14). Thus, even if we obtain the non-universal gaugino mass relation from the LHC and ILC, we may still have the gauge coupling unification at the GUT scale.

(4) Re$S = 0$, $F^\tau \neq 0$, $F^S = 0$.

This case is the same as the mSUGRA.

(5) Re$S = 0$, $F^\tau = 0$, $F^S \neq 0$.

In this case, the gauge coupling relation at the GUT scale is given by Eq. (1), while the gaugino mass relation is given by Eq. (12) or Eq. (14). Also, the gaugino mass relation for $a_i \neq 0$ is given by Eq. (16) as well.

III. GRAND UNIFIED THEORIES

In four-dimensional GUTs, the non-universal SM gauge kinetic function can be generated after GUT gauge symmetry breaking by the high-dimensional operators [7–18]. The generic gauge kinetic function in the superpotential is

$$W \supset \frac{1}{2}\mathrm{Tr}\left[W^a W^b \left(\tau\delta_{ab} + \lambda\frac{\Phi_{ab}}{M_*}S\right)\right] ~, \tag{17}$$

where λ is the Yukawa coupling constant, and Φ_{ab} transforms as the symmetric product of two adjoint representations. After Φ_{ab} obtains a VEV, we obtain the gauge kinetic functions in Eq. (9) where ϵ is the product of λ, the VEV of Φ_{ab}, and suitable normalization factors.

First, let us study the $SU(5)$ models. The symmetric product of the adjoint representation **24** of $SU(5)$ can be decomposed into irreducible representations of $SU(5)$ as follows

$$(\mathbf{24}\times\mathbf{24})_{\text{symmetric}} ~=~ \mathbf{1}\oplus\mathbf{24}\oplus\mathbf{75}\oplus\mathbf{200} ~. \tag{18}$$

We present a_i and index k for each irreducible representation in Table I. Thus, using our general formulae in Section II, we have all the GUT-scale gauge coupling relations and all the gaugino mass relations in $SU(5)$ models. Especially, in the traditional $SU(5)$ models that have been studied extensively so far, the GUT Higgs field is in the representation **24**, and then the index k is 5/3. By the way, the gaugino mass relations for the Higgs fields in the representations **24** and **75** have been studied previously [10].

$SU(5)$	a_1	a_2	a_3	k
1	1	1	1	0/0
24	$-1/2$	$-3/2$	1	5/3
75	-5	3	1	$-1/3$
200	10	2	1	1/9

TABLE I: a_i and k for each irreducible representation in $SU(5)$ models.

Second, let us consider the $SO(10)$ models. The symmetric product of the adjoint representation **45** of $SO(10)$ can be decomposed into irreducible representations of $SO(10)$ as follows

$$(\mathbf{45}\times\mathbf{45})_{\text{symmetric}} ~=~ \mathbf{1}\oplus\mathbf{54}\oplus\mathbf{210}\oplus\mathbf{770} ~. \tag{19}$$

The $SO(10)$ models can be broken down to the Georgi-Glashow $SU(5) \times U(1)$ models, the flipped $SU(5) \times U(1)_X$ models, and the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ models. We present a_i and indices k for each irreducible representation in Table II, Table III and Table IV for the $SO(10)$ models whose gauge symmetries are broken down to the Georgi-Glashow $SU(5) \times U(1)$ gauge symmetries, the flipped $SU(5) \times U(1)_X$ gauge symmetries, and the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries, respectively. We emphasize that our numbers a_i in Table II, Table III and Table IV are the same as the results obtained in the corresponding Tables in Ref. [15]. Thus, using our generical formulae in Section II, we have all the GUT-scale gauge coupling relations and all the gaugino mass relations in $SO(10)$ models.

In the traditional $SO(10)$ models that have been studied extensively so far, the GUT Higgs fields are in the representations **45** as well as **16** and $\overline{\mathbf{16}}$ [33]. Thus, the above discussions can not be applied directly. In this case, the discussions on the GUT-scale gauge coupling relation and the gaugino mass relation are similar to these in the field theory realization of the F-theory $SO(10)$ models with $U(1)_{B-L}$ flux. As discussed in the following, the index k in the traditional $SO(10)$ models is 5/3 as well.

$SO(10)$	$SU(5) \times U(1)$	a_1	a_2	a_3	k
1	$(\mathbf{1}, \mathbf{0})$	1	1	1	0/0
54	$(\mathbf{24}, \mathbf{0})$	$-1/2$	$-3/2$	1	5/3
210	$(\mathbf{1}, \mathbf{0})$	1	1	1	0/0
	$(\mathbf{24}, \mathbf{0})$	$-1/2$	$-3/2$	1	5/3
	$(\mathbf{75}, \mathbf{0})$	-5	3	1	$-1/3$
770	$(\mathbf{1}, \mathbf{0})$	1	1	1	0/0
	$(\mathbf{24}, \mathbf{0})$	$-1/2$	$-3/2$	1	5/3
	$(\mathbf{75}, \mathbf{0})$	-5	3	1	$-1/3$
	$(\mathbf{200}, \mathbf{0})$	10	2	1	1/9

TABLE II: a_i and k for each irreducible representation in $SO(10)$ models whose gauge symmetry is broken down to the Georgi-Glashow $SU(5) \times U(1)$ gauge symmetries.

$SO(10)$	$SU(5)\times U(1)_X$	a_1	a_2	a_3	k
$\mathbf{1}$	$(\mathbf{1},\mathbf{0})$	1	1	1	0/0
$\mathbf{54}$	$(\mathbf{24},\mathbf{0})$	$-1/2$	$-3/2$	1	5/3
	$(\mathbf{1},\mathbf{0})$	$-19/5$	1	1	0
$\mathbf{210}$	$(\mathbf{24},\mathbf{0})$	7/10	$-3/2$	1	25/3
	$(\mathbf{75},\mathbf{0})$	$-1/5$	3	1	$-5/3$
	$(\mathbf{1},\mathbf{0})$	77/5	1	1	0
$\mathbf{770}$	$(\mathbf{24},\mathbf{0})$	$-101/10$	$-3/2$	1	25/111
	$(\mathbf{75},\mathbf{0})$	$-1/5$	3	1	$-5/3$
	$(\mathbf{200},\mathbf{0})$	2/5	2	1	$-5/3$

TABLE III: a_i and k for each irreducible representation in $SO(10)$ models whose gauge symmetry is broken down to the flipped $SU(5)\times U(1)_X$ gauge symmetries.

$SO(10)$	$SU(4)_C\times SU(2)_L\times SU(2)_R$	a_1	a_2	a_3	k
$\mathbf{1}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1	1	1	0/0
$\mathbf{54}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$-1/2$	$-3/2$	1	5/3
	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$-3/5$	1	0	$-5/3$
$\mathbf{210}$	$(\mathbf{15},\mathbf{1},\mathbf{1})$	$-4/5$	0	1	5/9
	$(\mathbf{15},\mathbf{1},\mathbf{3})$	1	0	0	0
	$(\mathbf{1},\mathbf{1},\mathbf{1})$	19/10	5/2	1	5/3
$\mathbf{770}$	$(\mathbf{1},\mathbf{1},\mathbf{5})$	1	0	0	0
	$(\mathbf{15},\mathbf{1},\mathbf{3})$	1	0	0	0
	$(\mathbf{84},\mathbf{1},\mathbf{1})$	32/5	0	1	$-5/27$

TABLE IV: a_i and k for each irreducible representation in $SO(10)$ models whose gauge symmetry is broken down to the Pati-Salam $SU(4)_C\times SU(2)_L\times SU(2)_R$ gauge symmetries.

IV. F-THEORY GUTS

We consider the field theory realization of the $U(1)$ flux effects on the SM gauge kinetic functions in F-theory GUTs, and study their GUT-scale gauge coupling relations and their gaugino mass relations [24, 27–29]. In the F-theory $SU(5)$ models, the $SU(5)$ gauge symmetry is broken down to the SM gauge symmetry by turning on the $U(1)_Y$ flux. To realize the $U(1)_Y$ flux corrections to the SM gauge kinetic functions in the four-dimensional $SU(5)$ models, we consider the following superpotential term for the $SU(5)$ gauge kinetic function

$$W \supset \frac{1}{2}\mathrm{Tr}\left[W^a W^b \left(\tau\delta_{ab} + \left(\frac{Z^2\delta_{ab} + \lambda\Phi_a\Phi_b}{M_*^2}\right) S\right)\right] , \tag{20}$$

where Z is a SM singlet Higgs field, and Φ_a and Φ_b are the Higgs fields in the adjoint representation of $SU(5)$ which have VEVs along the $U(1)_Y$ direction. Five stacks of seven-branes give us $U(5)$ symmetry, thus, the Z^2 term is similar to the flux for the global $U(1)$ of $U(5)$, and the $\Phi_a\Phi_b$ term is similar to the $U(1)_Y$ flux [24, 27]. After $SU(5)$ gauge symmetry is broken down to the SM gauge symmetry, with suitable definition of ϵ, we obtain [24, 27]

$$a_1 = \frac{1}{2}\left(\alpha + \frac{6}{5}\right) \ , \ a_2 = \frac{1}{2}\left(\alpha + 2\right) \ , \ a_3 = \frac{1}{2}\alpha \ , \tag{21}$$

where α is a real number. In F-theory models, α should be quantized due to flux quantization. Thus, using Eqs. (11) and (12) or Eqs. (13) and (14), we can easily obtain the gauge coupling relation at the GUT scale and the gaugino mass relation whose index k is 5/3 [31].

In the F-theory $SO(10)$ models where the $SO(10)$ gauge symmetry is broken down to the flipped $SU(5) \times U(1)_X$ gauge symmetry by turning on the $U(1)_X$ flux [22, 23, 26, 28], we can show that the gauge kinetic functions for $SU(5)$ and $U(1)_X$ are exactly the same at the unification scale [28]. In the field theory realization, we consider the following superpotential term for the $SO(10)$ gauge kinetic function

$$W \supset \frac{1}{2}\mathrm{Tr}\left[W^a W^b \left(\tau\delta_{ab} + \lambda\frac{\Phi_a\Phi_b}{M_*^2} S\right)\right] \ , \tag{22}$$

where Φ_a and Φ_b are the Higgs fields in the adjoint representation of $SO(10)$. To have similar effects as the $U(1)_X$ flux, Φ_a and Φ_b obtain VEVs along the $U(1)_X$ direction. Thus, with suitable definition of ϵ, we get [28]

$$a_1 \ = \ a_2 \ = \ a_3 \ = \ 1 \ . \tag{23}$$

Therefore, similar to the mSUGRA, we obtain the gauge coupling unification at the GUT scale in Eq. (1) and the gaugino mass relation in Eq. (2), *i.e.*, we have $k = \infty$.

In the F-theory $SO(10)$ models, the $SO(10)$ gauge symmetry can also be broken down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry by turning on the $U(1)_{B-L}$

flux [25, 29]. To realize the $U(1)_{B-L}$ flux corrections to the SM gauge kinetic functions in four-dimensional $SO(10)$ models, we still consider the superpotential term in Eq. (22), where Φ_a and Φ_b obtain VEVs along the $U(1)_{B-L}$ direction. Thus, with suitable definition of ϵ, we get [29]

$$a_1 \;=\; \frac{2}{5}\;,\quad a_2 \;=\; 0\;,\quad a_3 \;=\; 1\;. \tag{24}$$

Therefore, using Eqs. (11) and (12) or Eqs. (13) and (14), we can easily obtain the gauge coupling relation at the GUT scale and the gaugino mass relation whose index k is 5/3 [31].

V. CONCLUSIONS

In GUTs with gravity mediated supersymmetry breaking, we considered the generic gauge coupling relations at the GUT scale, and the general gaugino mass relations which are valid from the GUT scale to the electroweak scale at one loop. Interestingly, the gauge coupling relations and the gaugino mass relations at the GUT-scale are given by the same equation, *i.e.*, $1/\alpha_i$ and M_i/α_i satisfy the same equation respectively for the gauge coupling relations and the gaugino mass relations. Thus, we define the index k for these relations. Because the index k can be calculated in GUTs and can be determined at the LHC and future ILC, we gave a concrete definition of the GUT scale in these theories, and suggested a new way to test general GUTs at the future experiments. We also dicussed five special scenarios with interesting possibilities. With our generic formulae, we presented all the GUT-scale gauge coupling relations and all the gaugino mass relations in the $SU(5)$ and $SO(10)$ models, and calculated the corresponding indices. In particular, the index k is 5/3 [10] in the traditional $SU(5)$ and $SO(10)$ models that have been studied extensively so far. Moreover, we studied the field theory realization of the $U(1)$ flux effects on the SM gauge kinetic functions in F-theory GUTs. We found that in the $SU(5)$ and $SO(10)$ models respectively with $U(1)_Y$ and $U(1)_{B-L}$ fluxes, the index k is 5/3, while in the $SO(10)$ models with $U(1)_X$ flux, the GUT-scale gauge coupling relation and gaugino mass relation are the same as these in mSUGRA. In short, the gaugino mass relation with index $k = 5/3$ [10] definitely deserve further detail study.

Acknowledgments

This research was supported in part by the DOE grant DE-FG03-95-Er-40917 (TL and DVN), by the Natural Science Foundation of China under grant No. 10821504 (TL), and by the Mitchell-Heep Chair in High Energy Physics (TL).

---

[1] J. R. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B **249**, 441 (1990); Phys. Lett. B **260**, 131 (1991); U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B **260**, 447 (1991); P. Langacker and M. X. Luo, Phys. Rev. D **44**, 817 (1991).

[2] J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos and M. Srednicki, Phys. Lett. B **127**, 233 (1983); J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive and M. Srednicki, Nucl. Phys. B **238**, 453 (1984).

[3] H. Goldberg, Phys. Rev. Lett. **50**, 1419 (1983) [Erratum-ibid. **103**, 099905 (2009)].

[4] A. H. Chamseddine, R. L. Arnowitt and P. Nath, Phys. Rev. Lett. **49**, 970 (1982); H. P. Nilles, Phys. Lett. B **115**, 193 (1982); L. E. Ibanez, Phys. Lett. B **118**, 73 (1982); R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B **119**, 343 (1982); H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **120**, 346 (1983); J. R. Ellis, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **121**, 123 (1983); J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **125**, 275 (1983); L. J. Hall, J. D. Lykken and S. Weinberg, Phys. Rev. D **27**, 2359 (1983).

[5] W. S. Cho, K. Choi, Y. G. Kim and C. B. Park, Phys. Rev. Lett. **100**, 171801 (2008); M. M. Nojiri, Y. Shimizu, S. Okada and K. Kawagoe, JHEP **0806**, 035 (2008).

[6] V. D. Barger, T. Han, T. Li and T. Plehn, Phys. Lett. B **475**, 342 (2000).

[7] C. T. Hill, Phys. Lett. B **135**, 47 (1984).

[8] Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52**, 875 (1984).

[9] J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B **247**, 373 (1984).

[10] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **155**, 381 (1985).

[11] M. Drees, Phys. Lett. B **158**, 409 (1985).

[12] G. Anderson, H. Baer, C. h. Chen and X. Tata, Phys. Rev. D **61**, 095005 (2000).

[13] N. Chamoun, C. S. Huang, C. Liu and X. H. Wu, Nucl. Phys. B **624**, 81 (2002).

[14] J. Chakrabortty and A. Raychaudhuri, Phys. Lett. B **673**, 57 (2009).

[15] S. P. Martin, Phys. Rev. D **79**, 095019 (2009).

[16] S. Bhattacharya and J. Chakrabortty, Phys. Rev. D **81**, 015007 (2010).

[17] D. Feldman, Z. Liu and P. Nath, Phys. Rev. D **80**, 015007 (2009).

[18] N. Chamoun, C. S. Huang, C. Liu and X. H. Wu, arXiv:0909.2374 [hep-ph].

[19] X. Calmet, S. D. H. Hsu and D. Reeb, Phys. Rev. Lett. **101**, 171802 (2008).

[20] C. Vafa, Nucl. Phys. B **469**, 403 (1996).

[21] R. Donagi and M. Wijnholt, arXiv:0802.2969 [hep-th].

[22] C. Beasley, J. J. Heckman and C. Vafa, JHEP **0901**, 058 (2009).

[23] C. Beasley, J. J. Heckman and C. Vafa, JHEP **0901**, 059 (2009).

[24] R. Donagi and M. Wijnholt, arXiv:0808.2223 [hep-th].

[25] A. Font and L. E. Ibanez, JHEP **0902**, 016 (2009).

[26] J. Jiang, T. Li, D. V. Nanopoulos and D. Xie, Phys. Lett. B **677**, 322 (2009).

[27] R. Blumenhagen, Phys. Rev. Lett. **102**, 071601 (2009).

[28] J. Jiang, T. Li, D. V. Nanopoulos and D. Xie, Nucl. Phys. B **830**, 195 (2010).

[29] T. Li, arXiv:0905.4563 [hep-th].

[30] G. K. Leontaris and N. D. Tracas, arXiv:0912.1557 [hep-ph].

[31] T. Li, J. A. Maxin and D. V. Nanopoulos, arXiv:1002.1031 [hep-ph].

[32] T. Li and D. V. Nanopoulos, in preparation.

[33] H. Georgi and D. V. Nanopoulos, Nucl. Phys. B **155**, 52 (1979).